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SO(32) Spinors of type I and other solitons on brane-antibrane pair

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Abstract: We construct the SO(32) spinor state in weakly coupled type I string theory as a kink solution of the tachyon field on the D-string – anti-D-string pair and calculate its mass. We also give a description of this system in terms of an exact boundary conformal field theory and show that in this description this state can be regarded as a non-supersymmetric D0-brane in type I string theory. This construction can be generalised to represent the D0-brane in type IIA string theory as a vortex solution of the tachyon field on the membrane anti-membrane pair, and the D-string of type I string theory as a topological soliton of the tachyon field on the D5-brane anti-D5-brane pair.

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1. Introduction and summary

It has been conjectured that SO(32) heterotic string theory is dual to type I string theory [1, 2, 3, 4]. Many tests of this conjectured duality have been performed, including comparison of the BPS spectrum in compactified string theories [5]. However, even in ten dimensions, the SO(32) heterotic string theory contains, in its perturbative spectrum, states which are stable but are not BPS. These are the states which transform as spinors of SO(32). The lightest states belonging to the spinor representation of SO(32) must be stable since they cannot decay into anything else. On the other hand they are not BPS states, since the theory in ten dimensions has only $N = 1$ supersymmetry and hence no central charge. Like BPS states, these stable non-BPS states must exist at all values of the coupling, although their mass is not determined by any known formula. Thus if the duality between the type I and SO(32) string theory is correct then we must find these states in the weakly coupled type I string theory as well. This is the problem we address in this paper.

A description of this state was proposed earlier in ref. [6] as a tachyonic kink solution on the D-string anti- D-string pair. In section 2 of the paper we discuss this construction in detail. It is well known that on the world volume of the D-string anti-D-string pair there is a tachyonic mode coming from open string with one end on the D-string and the other end on the anti- D-string [7, 8, 9, 10]. At the minimum of the tachyon potential, the negative contribution to the energy density from the tachyon potential exactly cancels the positive contribution from the tension of the D-string and the anti- D-string [6]. Since in type I theory the tachyon is a real field, and furthermore, the dynamics is invariant under a change of sign of the tachyon field, the minimum of the potential comes in pairs $\pm T_0$. Thus we can construct a tachyonic kink solution on the D-string anti- D-string pair interpolating between the degenerate vacua at $\pm T_0$. We show that this state carries SO(32) spinor quantum numbers. Furthermore its mass is finite since far away from the core of the soliton the energy density vanishes. Thus this is the ideal candidate for the state we are looking for.

The mass of this state can be easily shown to be proportional to the inverse coupling (g^{-1}) of the type I theory by a simple scaling argument. However computing the constant of proportionality directly requires the detailed knowledge of the tachyon potential which is not available to us. We use an indirect method for computing this coefficient by considering the case where the type I theory has been compactified on a circle of radius R, and the D-string anti- D-string pair is wrapped along that circle. We show that in order to get an SO(32) spinor state we must impose an anti-periodic boundary condition on the tachyon, and as a result the zero momentum mode of the tachyon is absent. As we reduce the radius of the circle, the contribution to the effective mass^2 of various modes from the momentum along the circle increases, and one finds that at a critical radius $R = (1/\sqrt{2})$ the tachyonic mode ceases to be tachyonic. In fact at this point the lowest momentum mode of the tachyon represents an exactly marginal deformation. We interprete this critical radius as the point where the SO(32) spinor state becomes degenerate with the trivial vacuum of the D-string anti- D-string pair where the tachyon vanishes. Since the mass of the wrapped D-string anti- D-string pair in the absence of tachyon vacuum expectation value (vev) can be easily computed in the weak coupling limit, this gives an indirect measurement of the mass of the spinor state. Although this gives us the mass only at the critical radius, we give a heuristic argument showing that the mass is independent of the radius and hence also represents the mass of the spinor state at infinite radius.

This 'target space viewpoint' gives an intuitive description of the SO(32) spinor state; however it does not give us an exact conformal field theory description of the system. This is the problem we address in sections 3 and 4. In section 3 we work at the critical radius $R = (1/\sqrt{2})$, and study the effect of switching on the marginal deformation associated with 'tachyon' condensation. Parametrizing the tachyon vev at this critical radius by an appropriate parameter α we compute the spectrum of open string states on the D-string anti- D-string system at arbitrary α. In particular we find that α is a periodic variable with periodicity 2. At any value of α other than zero (mod 2) the mode representing the freedom of separating the string anti-string pair becomes massive, showing that the pair is bound. In fact the mass of this mode is maximum at $\alpha = 1$ showing that at this point the pair is maximally bound. Also for all $\alpha \neq 0$ there is a zero mode representing the freedom of translating the solution along the compact direction, indicating the fact that the translation invariance of the original string anti-string configuration along the compact direction is broken as we switch on the tachyon vev.

At $R = (1/\sqrt{2})$ all values of α represent configurations of same mass, and hence we cannot unambiguously identify the point which corresponds to the spinor state, *i.e.* which will correspond to the minimum of the potential when we increase R beyond this critical radius. This is the problem we address in section 4 by studying the effect of switching on the perturbation in the closed string sector corresponding to the radius deformation. We find that the 'tachyonic mode' develops a one point function at all α except at $\alpha = 0$ and 1. Thus only these two points correspond to the extremum of the tachyon potential. Furthermore, the 'tachyonic mode' becomes tachyonic at $\alpha = 0$ as expected, but acquires positive mass^2 at $\alpha = 1$. This shows that the point $\alpha = 1$ represents a stable minimum of the potential, and hence represents the SO(32) spinor state that we have been looking for.

One can now examine the limit $R \to \infty$. This is best studied using a different set of world-sheet bosonic and fermionic fields, related to the original bosonic and fermionic fields representing cooordinate along the compact direction by a series of bosonization, fermionization and duality transformation. The net result is that in terms of the new bosonic coordinate, the soliton at $\alpha = 1$ can be represented as a D0-brane, with Dirichlet boundary condition in all directions, and the radius of the new boson goes to ∞ as $R \to \infty$. This gives an exact world-sheet description of the SO(32) spinor state of type I string theory and allows us to calculate its mass. This is same as the mass calculated in section 2. The R independence of the mass. which was a crucial assumption in the analysis of section 2, can be easily understood by noting that the mass of a D0-brane situated on a compact circle does not depend on the radius of the circle.

In section 5 we consider other solitons on brane anti-brane system. The first example we study is a membrane anti-membrane pair of type IIA string theory. There is a complex tachyon field living on this system, and hence we expect its vacuum manifold (minimum of the potential) to be a circle S^1. Since $\pi_1(S^1)$ is non-trivial, we can now construct topologically stable tachyonic vortex solution on the world-volume of the membrane anti-membrane pair. By analysing the quantum numbers carried by this soliton we can identify it as the D0-brane of type IIA string theory. This construction can be easily generalized to represent a Dp-brane of type II string theory as a vortex solution on the D-$(p+2)$-brane anti- D-$(p+2)$-brane pair.

The other example that we study in this section is that of a D5-brane anti- D5-brane pair in type I string theory. The tachyon field living on this system is represented by a 2×2 matrix, and the vacuum manifold for this tachyon field can be shown to be S^3. Thus we can construct a topologically stable string like solution on this $(5+1)$ dimensional world volume with the property that the asymptotic boundary of the string, which in this case is S^3, wraps around the vacuum manifold with unit winding number. A detailed analysis of this solution shows that it carries the same quantum numbers as those of a D-string of type I string theory. Thus it gives a different representation of the D-string in this theory.

Our analysis also throws light on a related problem discussed in refs. [11, 12]. If we consider an orbifold plane of type IIB string theory of the form $R^{5,1} \times (R^4/(-1)^{F_L} \cdot \mathcal{I}_4)$, where $\mathcal{I}_4$ denotes the Z_2 transformation that reverses the sign of all coordinates along R^4 and $(-1)^{F_L}$ denotes the reversal of sign of all Ramond sector states on the left, then the twisted sector states contain a massless U(1) gauge field living on the orbifold plane $R^{(5,1)}$ located at the origin of R^4. It is a prediction of S-duality that there must be stable non-BPS states living on the orbifold plane which are charged under this U(1) gauge field [13]. In ref. [11] this state was identified as a half soliton living on a D-string anti- D-string pair transverse to the orbifold plane, obtained by modding out the kink solution described in section 2 (regarded as a configuration in type IIB string theory rather then in type I theory) by a reflection around its origin. On the other hand, ref. [12] attempted to describe the same state as a non-supersymmetric D0-brane, and constructed the boundary state describing this system. Our present analysis shows that these are different descriptions of the same physical object.[1]

We conclude this section by mentioning some related developments which have taken place during the recent years. Various examples of tachyon condensation in string theory have been studied in [14]. Stable non-BPS states in the context of supersymmetric field theory have been analyzed in [15, 16, 17]. In particular, [15] discusses non-BPS states carrying Z_2 quantum numbers similar to the states considered here. Other aspects of tachyons in non-supersymmetric string theories are discussed in [18].

2. SO(32) Spinor state as a tachyonic kink on the D-string anti-D-string pair

Let us consider type I string theory compactified on a circle S^1 of radius R and a D-string of type I wrapped on S^1. It is known that the world volume theory on the D-string is identical to that of the fundamental heterotic string in SO(32) heterotic string theory [4]. In particular the U(1) gauge field on the D-string is projected out by the requirement of world-sheet parity invariance. There is however a Z_2 subgroup of this U(1) which survives this projection, and hence we can introduce a Z_2 Wilson line on the S^1. Since the end of a fundamental string ending on the D-string is charged under the Z_2, in the presence of a Wilson line the wave function of an open string with one end on the D-string will pick up an extra $-$ sign under a $2\pi R$ translation along S^1.

The open string states with one end on the D-string and the other end on any of the 32 9-branes filling up the space-time provide the 32 fermionic degrees of freedom on the D-string world-volume. Due to the possibility of putting the Z_2 Wilson line on the D-string, these fermions can satisfy either periodic or anti-periodic bounday condition along S^1. If we put anti-periodic boundary condition, then there are no fermion zero

[1]The apparent discrepancy in the mass of the state calculated in the two approaches [12] seems to be due to the different units that have been used in the two papers.

modes, and the states are in the scalar conjugacy class of SO(32). On the other hand if we put periodic boundary condition on these fermions, then there are zero modes of the 32 fermions, and the quantization of these zero modes give a ground state (and hence all excited states) in the spinor conjugacy class of SO(32).

Thus if we consider a system of a D-string – anti- D-string[2] pair of type I, both wrapped on S^1, and put Z_2 Wilson line on one of them, the state will belong to the spinor conjugacy class of SO(32), and will not carry any other conserved charge as the Ramond-Ramond (RR) two form charges of the D-string and the anti- D-string will cancel each other. Thus this state has the correct quantum numbers, and we expect the lowest mass state in this sector to represent the lightest SO(32) spinor state of type I string theory. Naively, the mass of this state is given by,

$$2 \cdot 2\pi R \cdot T_D \,, \tag{2.1}$$

where T_D represents the tension of a single D-string:

$$T_D = \frac{1}{2\pi g} \,, \tag{2.2}$$

g being the string coupling constant. (We are working in units where we have set $\alpha' = 1$. Also we shall view type I string theory as the result of modding out type IIB theory by the world-sheet parity transformation Ω, and use type IIB units before projection under Ω to calculate all masses.) From eq. (2.1) we see that as $R \to \infty$, the mass of the D-string anti-D-string pair goes to infinity. Thus although this configuration has the right quantum numbers, it cannot represent the state we are looking for.

The situation improves upon noting that on the D-string anti- D-string world volume there is a tachyon field [7, 8, 9, 10], representing the fact that we are actually sitting at the top of a potential well. Thus in order to find the lowest energy configuration with these quantum numbers we must allow the tachyon field to acquire a vacuum expectation value and go to the minimum of its potential. Let T denote the tachyon field on the D-string anti- D-string world-volume, $g^{-1}V(T)$ denote the tachyon potential,[3] and T_0 be the point where $V(T)$ is at its minimum. It was argued in [6] that,

$$g^{-1}V(T_0) + 2T_D = 0 \,, \tag{2.3}$$

so that at the minimum of the tachyon potential the net energy per unit length of the D-string anti- D-string pair vanishes. Note that since the tachyon arises in the open string sector with one end on the D-string and the other end on the anti- D-string, it is odd under the Z_2 gauge transformation associated with either of the strings, and hence $V(T)$ is symmetric under $T \to -T$. Thus the locations of the minimum of $V(T)$ come in pairs $\pm T_0$.

[2]An anti- D-string is simply a D-string with opposite orientation.

[3]We normalise various fields so that the world-volume action on the D-string has an overall factor of g^{-1} in front, and does not have any other g dependence.

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Naively one would think that the ground state of the D-string anti- D-string system that we have been considering will correspond to a constant tachyon configuration $T(x) = T_0$, where x denotes the coordinate along S^1. However since we have put a Z_2 Wilson line on one of the D-strings, and since the tachyon is odd under this Z_2 gauge transformation, it must be anti-periodic under translation by $2\pi R$ along S^1. This shows that $T = T_0$ is not an allowed configuration. Instead, for large R, the minimum energy configuration will correspond to a kink solution such that[4]

$$T(x) \to -T_0 \quad \text{for} \quad x << 0\,, \qquad T(x) \to T_0 \quad \text{for} \quad x >> 0\,, \tag{2.4}$$

so that the tachyon satisfies the anti-periodic boundary condition:

$$T(\pi R) = -T(-\pi R)\,. \tag{2.5}$$

This configuration has finite mass in the $R \to \infty$ limit as a consequence of (2.3) since the energy density vanishes far away from the core. In fact, far away from the core the D-string anti- D-string configuration is indistinguishible from the vacuum since it does not carry any net charge or energy density, so the configuration is localised near $x = 0$ and represents *a particle like state* in the ten dimensional type I string theory. The total mass of this state can be computed by integrating the energy density along x. Since the dependence on the string coupling g of this energy density is through an overall multiplicative factor of g^{-1}, we can immediately conclude that the mass of this soliton is given by

$$m_{spinor} = C/g\,, \tag{2.6}$$

where C is a numerical constant. Explicit computation of C requires a detailed knowledge of the tachyon potential, but we shall compute it through an indirect argument later.

Thus we see that *the SO(32) spinor state in type I string theory can be identified as the tachyonic kink on the D-string anti- D-string pair.* Although our argument based on circle compactification already shows that this state transforms in the spinor representation of SO(32), we can also see it directly without the circle compactification.[5] On the world-volume of the D-string there are 32 left-moving Majorana fermions $\chi^i_{(1)}$, and on the world volume of the anti- D-string there are 32 right-moving Majorana fermions $\chi^i_{(2)}$ ($1 \le i \le 32$). Since under the $Z_2 \times Z_2'$ gauge symmetry on the world volume of the D-string anti- D-string system the fields $\chi^i_{(1)}$, $\chi^i_{(2)}$ and T carry charges $(-1, 1)$, $(1, -1)$ and $(-1, -1)$ respectively, the coupling between these fields must have the form:

$$L_{int} = i f(T) \chi^i_{(1)} \chi^i_{(2)} + \cdots\,, \tag{2.7}$$

where $f(T)$ is an odd function of T. $\cdots$ in the above equation denote terms quartic and higher order in the fermion fields. Thus the relevant quadratic part of the fermionic

[4]Here we have taken the core of the soliton to be located at $x = 0$.

[5]The following investigation was suggested by N. Seiberg.

lagrangian is given by:

$$L_{tot} = i[\chi^i_{(1)}(\partial_t - \partial_x)\chi^i_{(1)} + \chi^i_{(2)}(\partial_t + \partial_x)\chi^i_{(2)}] + if(T)\chi^i_{(1)}\chi^i_{(2)} \,. \tag{2.8}$$

In general the kinetic terms could be renormalized by even functions of T, but these could be absorbed into a redefinition of the fields $\chi^i_{(1)}$ and $\chi^i_{(2)}$. We shall now look for fermion zero modes by looking at the time independent equations of motion derived from the lagrangian (2.8):

$$\partial_x \begin{pmatrix} \chi^i_{(1)} \\ \chi^i_{(2)} \end{pmatrix} = f(T)\sigma_1 \begin{pmatrix} \chi^i_{(1)} \\ \chi^i_{(2)} \end{pmatrix} \,, \tag{2.9}$$

where σ_i denote the Pauli matrices:

$$\sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \qquad \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \qquad \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{2.10}$$

Let us now assume that $f(T_0) > 0$. Since $f(T)$ is an odd function of T, this would imply that $f(-T_0) < 0$. Using this we get the following normalizable solutions of eq. (2.9)

$$\begin{pmatrix} \chi^i_{(1)} \\ \chi^i_{(2)} \end{pmatrix} = \chi^i_0 \exp(\sigma_1 \int_0^x f(T(x'))dx') \begin{pmatrix} 1 \\ -1 \end{pmatrix} \,. \tag{2.11}$$

χ^i_0 are independent constants. If $f(T_0) < 0$, then the vector $\begin{pmatrix} 1 \\ -1 \end{pmatrix}$ in (2.11) is replaced by $\begin{pmatrix} 1 \\ 1 \end{pmatrix}$. In either case we get 32 fermionic zero modes χ^i_0 transforming in the vector representation of SO(32). Quantization of these zero modes gives states in the spinor as well as in the conjugate spinor representation of SO(32). However, since the zero modes are odd under the combined action of Z_2 and Z'_2 generators which leave T and hence the background solution invariant, by demanding that the states are invariant under this diagonal Z_2 group leaves us only with states in the spinor representation of the SO(32) group. This shows that the tachyonic kink solution indeed has the right quantum numbers.

Let us now turn to the indirect argument determining the value of the numerical coefficient C appearing in the mass formula (2.6). For this we start with type I theory on a circle S^1 of radius R, and consider the original configuration, namely a D-string anti- D-string pair wrapped on S^1 with a Z_2 Wilson line on one of the strings. Since the tachyon field living on the world volume of this system must satisfy anti-periodic boundary condition along S^1, it admits a mode expansion of the form:

$$T(x,t) = \sum_{n\in Z} T_{n+\frac{1}{2}}(t) e^{i\frac{n+\frac{1}{2}}{R}x} \,. \tag{2.12}$$

Note that the zero momentum mode is absent. The effective mass^2 of $T_{\pm(n+\frac{1}{2})}$ is given by

$$m_n^2 = \frac{(n+\frac{1}{2})^2}{R^2} - \frac{1}{2} \,, \tag{2.13}$$

where $-1/2$ is the mass^2 of the tachyon field on the infinite D-string anti- D-string pair. From eq. (2.13) we see that $m_n^2 \geq 0$ for all n if

$$R \leq \frac{1}{\sqrt{2}}\,. \tag{2.14}$$

In particular this shows that at $R = \frac{1}{\sqrt{2}}$ the modes $T_{\pm\frac{1}{2}}$ represent marginal deformation. In fact as will be shown in the next section, this mode actually represents an exactly marginal deformation. Thus we arrive at the following intuitive picture.

- For $R > (1/\sqrt{2})$ the total mass of the D-string anti- D-string pair before tachyon condensation is larger than the mass m_{spinor} of the spinor state, signalling the presence of a tachyonic mode in the D-string anti- D-string system:

$$4\pi R T_D > m_{spinor} \qquad \text{for} \qquad R > \frac{1}{\sqrt{2}}\,. \tag{2.15}$$

- For $R < (1/\sqrt{2})$ the total mass of the D-string anti- D-string pair before tachyon condensation is smaller than the mass m_{spinor} of the spinor state, signalling the absence of a tachyonic mode in the D-string anti- D-string system:

$$4\pi R T_D < m_{spinor} \qquad \text{for} \qquad R < \frac{1}{\sqrt{2}}\,. \tag{2.16}$$

  This also shows that in this range of R the spinor state becomes unstable against decay into a D-string anti- D-string pair.

- For $R = (1/\sqrt{2})$ the total mass of the D-string anti- D-string pair before tachyon condensation is equal to the mass m_{spinor} of the spinor state, signalling the presence of a marginal deformation connecting the two states.

$$4\pi R T_D = m_{spinor} \qquad \text{for} \qquad R = \frac{1}{\sqrt{2}}\,. \tag{2.17}$$

This gives

$$m_{spinor} = \sqrt{2}/g\,. \tag{2.18}$$

Comparing with eq. (2.6) we get

$$C = \sqrt{2}\,. \tag{2.19}$$

The above analysis determines the value of m_{spinor} at $R = (1/\sqrt{2})$. Since we are interested in determining m_{spinor} for $R = \infty$, we have implicitly assumed that m_{spinor} does not depend on R, *i.e.* it does not get affected by compactification of the theory on a circle. We shall now give an indirect argument in favour of this, and leave a more detailed analysis of this question to section 4. In order to study the effect of compactification on the mass of the spinor state we can estimate the interaction energy between the spinor and its (infinite set of) images situated at distance $2\pi n R$ on the

X-axis for all integer n. In particular the gravitational interaction in this case is of order

$$g^2 \cdot g^{-1} \cdot g^{-1} \sim 1\,, \tag{2.20}$$

where the g^2 factor arises from the Newton's constant and the factors of g^{-1} arise from the masses of each state in the interacting pair. Since the right hand side of (2.20) is much smaller than g^{-1} – the mass of the spinor state as computed earlier – we expect that the gravitational interaction between the spinor and its images will not significantly modify the mass of this state. Similar argument can be given for electromagnetic interactions as well. Thus we conclude that the mass of the spinor state is not significantly modified due to interaction with its images, and hence (2.18) represents the mass of this state in the infinite radius limit as well.

Finally, note that although we would have expected the classical soliton representing the SO(32) spinor state in (9+1) dimension to be invariant under the SO(9) rotation group, the soliton that we have constructed only has manifest SO(8) symmetry since the direction along which the original D-string anti- D-string pair lies is somewhat special. However, we shall see in section 4 that the boundary conformal field theory describing this state does have the full SO(9) rotational invariance.

3. Conformal field theory at the critical radius

Let X denote the scalar field corresponding to coordinate x along the S^1 and ψ, $\widetilde{\psi}$ its right- and left-moving fermionic partners on the world-sheet of the fundamental string. At this particular radius the conformal field theory of the bosonic field X is equivalent to that of a pair of right-moving fermionic fields ξ, η and a pair of left-moving fermion fields $\widetilde{\xi}$, $\widetilde{\eta}$ [19]. If we decompose X into its left- and right-moving parts as

$$X = X_L + X_R\,, \tag{3.1}$$

then the Bose-Fermi relation takes the form

$$e^{i\sqrt{2}X_R} = \frac{1}{\sqrt{2}}(\xi + i\eta)\,, \qquad e^{i\sqrt{2}X_L} = \frac{1}{\sqrt{2}}(\widetilde{\xi} + i\widetilde{\eta})\,. \tag{3.2}$$

We can find another representation of the same conformal field theory by rebosonising the fermions as follows:

$$\frac{1}{\sqrt{2}}(\xi + i\psi) = e^{i\sqrt{2}\phi_R}\,, \qquad \frac{1}{\sqrt{2}}(\widetilde{\xi} + i\widetilde{\psi}) = e^{i\sqrt{2}\phi_L}\,. \tag{3.3}$$

ϕ represents a free bosonic field with radius $1/\sqrt{2}$. There is a third representation in which we use a slightly different rebosonization:

$$\frac{1}{\sqrt{2}}(\eta + i\psi) = e^{i\sqrt{2}\phi'_R}\,, \qquad \frac{1}{\sqrt{2}}(\widetilde{\eta} + i\widetilde{\psi}) = e^{i\sqrt{2}\phi'_L}\,, \tag{3.4}$$

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where ϕ' is another scalar field of radius $1/\sqrt{2}$. For later use we list here the operator product expansions, and the relations between the currents of free fermions and bosons:

$$\psi(z)\psi(w) \simeq \xi(z)\xi(w) \simeq \eta(z)\eta(w) \simeq \frac{i}{z-w}\,, \tag{3.5}$$

$$\partial X_R(z)\partial X_R(w) \simeq \partial\phi_R(z)\partial\phi_R(w) \simeq \partial\phi'_R(z)\partial\phi'_R(w) \simeq -\frac{1}{2(z-w)^2}\,, \tag{3.6}$$

$$\psi\xi = i\sqrt{2}\partial\phi_R\,, \qquad \eta\xi = i\sqrt{2}\partial X_R\,, \qquad \psi\eta = i\sqrt{2}\partial\phi'_R\,. \tag{3.7}$$

Here $\simeq$ denotes equality up to non-singular terms. There are also similar relations involving the left-moving currents.

Since X satisfies Neumann boundary condition $X_L = X_R$ at the boundary of the world sheet, this translates to the Neumann boundary condition on the fermions:[6]

$$\xi_B = \widetilde{\xi}_B\,, \qquad \eta_B = \widetilde{\eta}_B\,, \tag{3.8}$$

where the subscript B denotes the boundary values of the fields. On the other hand in the NS sector, ψ satisfies the boundary condition

$$\psi_B = \widetilde{\psi}_B\,, \tag{3.9}$$

at both boundaries. Thus we see from (3.4) that ϕ and ϕ' both satisfy Neumann boundary condition at both ends. However, in the Ramond sector, at one of the two ends of the open string, ψ satisfies

$$\psi_B = -\widetilde{\psi}_B\,. \tag{3.10}$$

This translates to the fact that ϕ and ϕ' satisfy Dirichlet boundary condition at one end and Neumann boundary condition at the other end.

Up to overall numerical factors, the vertex operators corresponding to the states $T_{\pm\frac{1}{2}}$ are given in the -1 picture as [20]:

$$V_{\pm}^{(-1)} = \mp i e^{-\Phi_B} e^{\pm(i/\sqrt{2})X_B} \otimes \sigma_1\,, \tag{3.11}$$

and in the zero picture as

$$V_{\pm}^{(0)} = \psi_B e^{\pm(i/\sqrt{2})X_B} \otimes \sigma_1\,. \tag{3.12}$$

Φ denotes the bosonized ghost [20]. σ_1 is the 2×2 Chan-Paton factor representing that the tachyon originates in the open string sector with two ends of the open string on two different strings. The overall multiplicative factor of ∓ 1 in (3.11) is a matter of convention. Let

$$V_T = \frac{1}{\sqrt{2}}(V_+ + V_-)\,, \tag{3.13}$$

[6]These boundary conditions are written in the coordinate system where the open string world sheet is represented as the upper half plane.

represent vertex operators for the real component of $T_{1/2}$.[7] Using eqs.(3.2)-(3.13) we get

$$V_T^{(-1)} = e^{-\Phi_B}\eta_B \otimes \sigma_1, \qquad V_T^{(0)} = \psi_B \xi_B \otimes \sigma_1 \,. \tag{3.14}$$

We can now take ϕ, η and $\tilde{\eta}$ as independent fields, and represent the tachyon vertex operator in the zero picture as:

$$V_T^{(0)} = \frac{i}{\sqrt{2}}\partial\phi_B \otimes \sigma_1 \,, \tag{3.15}$$

for Neumann boundary condition on ϕ. ∂ denotes tangential derivative along the boundary. This shows that switching on the tachyon vev corresponds to switching on a U(1) Wilson line along the bosonic direction ϕ. On the other hand, for Dirichlet boundary condition on ϕ, we may rewrite (3.14) as

$$V_T^{(0)} = \frac{i}{\sqrt{2}}(\partial\phi_R - \partial\phi_L)_B \otimes \sigma_1 \,. \tag{3.16}$$

Switching on the tachyon vev now corresponds to changing the boundary condition from $\phi = 0$ to $\phi =$constant. Clearly, both (3.15) and (3.16) represent marginal deformation.

Before we study the effect of switching on such a tachyon vev on the spectrum of open strings, let us study the effect of various projections on the open string spectrum before switching on the tachyon vev. There are several discrete transformations under which the open string states are required to be invariant. Each such transformation includes transformation on the fields at the boundary, on the SL(2,R) invariant vacuum in the NS sector, as well as on the Chan-Paton factor which we shall represent by a 2×2 matrix Λ. These are as follows:

$$\begin{aligned}
(-1)^F &: X_B \to X_B \,, \quad \psi_B \to -\psi_B \,, \quad X_B^\mu \to X_B^\mu \,, \quad \psi_B^\mu \to -\psi_B^\mu \,, \\
&\quad |0\rangle \to -|0\rangle \,, \quad \Lambda \to \sigma_3 \Lambda \sigma_3 , \\
h &: X_B \to X_B + \frac{2\pi}{\sqrt{2}} , \quad \psi_B \to \psi_B \,, \quad X_B^\mu \to X_B^\mu \,, \quad \psi_B^\mu \to \psi_B^\mu \,, \\
&\quad |0\rangle \to |0\rangle \,, \quad \Lambda \to \sigma_3 \Lambda \sigma_3 \,,
\end{aligned} \tag{3.17}$$

where x^μ ($1 \le \mu \le 8$) denote the non-compact directions transverse to the string. In $(-1)^F$ the conjugation by σ_3 reflects the fact that the open strings with two ends on two different strings, represented by off diagonal Chan Paton matrices, have opposite GSO projection compared to open strings with both ends on the same string. The $-$ sign in the transformation law of the vacuum represents the fact that the NS sector ground state for open strings with both ends on the same string is odd under $(-1)^F$.

[7] We could have chosen to work with any particular direction in the complex $T_{1/2}$ plane; we choose the real direction for definiteness.

h represents translation by $2\pi/\sqrt{2}$ along x, and the conjugation of Λ by σ_3 in the transformation law of h is a reflection of the Z_2 Wilson line present along one of the strings which make open strings with off-diagonal Chan-Paton factors anti-periodic along x instead of peridic. Using eqs.(3.2) we see that the action of these transformations on the fields ξ_B and η_B at the boundary are given by:

$$\begin{aligned} (-1)^F \quad &: \quad \xi_B \to \xi_B\,, \quad \eta_B \to \eta_B\,, \\ h \quad &: \quad \xi_B \to -\xi_B\,, \quad \eta_B \to -\eta_B \end{aligned} \tag{3.18}$$

and hence, using eq. (3.3), (3.4) we get the following action on ϕ, ϕ':

$$\begin{aligned} (-1)^F \quad &: \quad \phi_B \to -\phi_B\,, \quad \phi'_B \to -\phi'_B\,, \\ h \quad &: \quad \phi_B \to \tfrac{2\pi}{\sqrt{2}} - \phi_B\,, \quad \phi'_B \to \tfrac{2\pi}{\sqrt{2}} - \phi'_B\,. \end{aligned} \tag{3.19}$$

Besides $(-1)^F$ and h, the states are also required to be invariant under the world-sheet parity transformation Ω which we have not listed here.

Let us now study the effect of switching on the tachyon vev. Since we have seen that the Ramond sector of the open strings correspond to putting Neumann boundary condition at one end and Dirichlet boundary condition at the other, and hence are forced to carry zero momentum and winding along ϕ, we see immediately that the tachyon vev, representing translation along ϕ at the Dirichlet end, and Wilson line along ϕ at the Neumann end, does not affect the Ramond sector states. On the NS sector states with Neumann boundary condition at both ends, the tachyon vev corresponds to switching on a Wilson line along ϕ. Let us parametrize this Wilson line by

$$\exp(i\int_0^{2\pi/\sqrt{2}} A_\phi d\phi) = \exp(i\alpha\pi\sigma_1/2)\,, \tag{3.20}$$

corresponding to

$$A_\phi = \frac{\alpha}{2\sqrt{2}}\sigma_1\,. \tag{3.21}$$

α is a parameter labelling the strength of the tachyon vev. Typically the effect of switching on the Wilson line is to shift the quantization rule for the momentum along ϕ for states which are charged under the corresponding gauge field. In particular, for a state carrying q units of charge under σ_1, the ϕ momentum gets shifted by an amount

$$\frac{\alpha}{2\sqrt{2}}q\,. \tag{3.22}$$

Since the 2×2 identity matrix I and the matrix σ_1 commute with the Chan-Paton factor σ_1 associated with the tachyon vertex operator, we see that states with Chan Paton factors proportional to I or σ_1 are neutral under the Wilson line and hence their masses are not affected by the tachyon vev. This leaves us with the NS sector states with Chan Paton factors σ_3 and σ_2 respectively. To study the effect of the tachyon vev on states from these sectors, let us note that the Chan Paton factors:

$$\sigma_3 \mp i\sigma_2 \tag{3.23}$$

carry charges ± 2 respectively under the generator σ_1. In the presence of the tachyon vev (3.20) the ϕ momentum quantization rule of these states get shifted by an amount:

$$\pm\frac{\alpha}{\sqrt{2}}\,. \tag{3.24}$$

Since ϕ has radius $(1/\sqrt{2})$, this shows that α is a periodic variable with period 2. It will be more convenient for our analysis to use the fermionic language where we use the fermionic fields ψ_B, ξ_B and η_B as independent boundary fields. Defining

$$\chi_B = \frac{1}{\sqrt{2}}(\xi_B + i\psi_B) = e^{i\phi_B/\sqrt{2}}\,, \tag{3.25}$$

we can represent the effect of the tachyon vev in the sectors $\sigma_3 \pm i\sigma_2$ by taking the mode expansions of χ_B, $\chi_B^\dagger$ as follows:

$$\begin{aligned}\chi_B &= \sum_{n\in Z} \chi_{n+\frac{1}{2}\pm\alpha} e^{-i\pi(n+\frac{1}{2}\pm\alpha)\tau}\\ \chi_B^\dagger &= \sum_{n\in Z} \chi^\dagger_{n+\frac{1}{2}\mp\alpha} e^{-i\pi(n+\frac{1}{2}\mp\alpha)\tau}\,.\end{aligned} \tag{3.26}$$

Note that in our notation

$$(\chi_{n+\frac{1}{2}\pm\alpha})^\dagger = \chi^\dagger_{-n-\frac{1}{2}\mp\alpha}\,. \tag{3.27}$$

The non-trivial anti-commutators are:

$$\{\chi_{n+\frac{1}{2}\pm\alpha}, \chi^\dagger_{-m-\frac{1}{2}\mp\alpha}\} = \delta_{mn}\,. \tag{3.28}$$

In the Fock space built up from these oscillators, we need to project onto states invariant under $(-1)^F$, h and Ω. From (3.17) we see that both $(-1)^F$ and h exchange the Chan Paton factors $\sigma_3 \mp i\sigma_2$. Since Ω acts on the Chan Paton factor by transposition, it also exchanges the two sectors. Due to this reason it is more convenient to take the independent generators to be $(-1)^F$, $(-1)^F h$ and $(-1)^F\Omega$. Since $(-1)^F h$ and $(-1)^F\Omega$ leave the Chan Paton factors untouched, we can impose invariance under these transformations on the Fock space states, and then take appropriate linear combinations of the states from the two sectors to get states invariant under $(-1)^F$. Let us for definiteness focus on the sector with Chan Paton factor $\sigma_3 + i\sigma_2$. The simplest way to implement invariance under $(-1)^F h$ and $(-1)^F\Omega$ is to start from states at $\alpha = 0$ satisfying these projections and then simply follow them as we continuously deform α. During this process the oscillators $\chi_{n+\frac{1}{2}}$ and $\chi^\dagger_{n+\frac{1}{2}}$ get deformed to $\chi_{n+\frac{1}{2}+\alpha}$ and $\chi^\dagger_{n+\frac{1}{2}-\alpha}$ respectively, and the Fock vacuum $|0\rangle$ get deformed to $|0\rangle_\alpha$ satisfying

$$\chi_{n+\frac{1}{2}+\alpha}|0\rangle_\alpha = 0, \qquad \chi^\dagger_{n+\frac{1}{2}-\alpha}|0\rangle_\alpha = 0, \qquad \text{for} \quad n \geq 0\,. \tag{3.29}$$

Notice that as α passes through $(1/2)$, the state $|0\rangle_\alpha$ is no longer the state with lowest L_0 eigenvalue. Instead for $(3/2) > \alpha > (1/2)$ the state with lowest L_0 eigenvalue, denoted by $|G\rangle$, is related to $|0\rangle_\alpha$ by the relation:

$$|0\rangle_\alpha = \chi^\dagger_{\frac{1}{2}-\alpha}|G\rangle\,, \tag{3.30}$$

Note that $|G\rangle$ has opposite $(-1)^F h$ charge compared to $|0\rangle$ since χ, $\chi^\dagger$ are odd under $(-1)^F h$.

Once we have determined the spectrum in each sector invariant under $(-1)^F h$ and $(-1)^F \Omega$, we can find $(-1)^F$ invariant combination of states on the two sectors by noting that under this transformation:

$$\begin{aligned}(\sigma_3 + i\sigma_2) &\to (\sigma_3 - i\sigma_2) \\ |0\rangle_\alpha &\to -|0\rangle_{-\alpha} \\ \chi_{n+\frac{1}{2}+\alpha} &\to \chi^\dagger_{n+\frac{1}{2}+\alpha} \\ \chi^\dagger_{n+\frac{1}{2}-\alpha} &\to \chi_{n+\frac{1}{2}-\alpha}\,. \end{aligned} \tag{3.31}$$

We now note some important features of the spectrum.

- At $\alpha = 1$ the fermions χ, $\chi^\dagger$ are again half integer moded. However, since the ground state $|G\rangle$ has opposite $(-1)^F h$ charge compared to the vacuum $|0\rangle$ at $\alpha = 0$, this point is not equivalent to $\alpha = 0$. In particular, if we denote by X^μ, ψ^μ ($1 \le \mu \le 8$) the bosonic coordinates and their fermionic partners transverse to the D-string, then the state $\psi^\mu_{-\frac{1}{2}}|G\rangle$ is odd under $(-1)^F h$ and is projected out. On the other hand, if we start from the state $\psi^\mu_{-\frac{1}{2}}|0\rangle$ at $\alpha = 0$ and follow it as α changes from 0 to 1, this state gets mapped to $\psi^\mu_{-\frac{1}{2}}\chi^\dagger_{-\frac{1}{2}}|G\rangle$, and has dimension 1. Thus there are no zero modes corresponding to separating the D-string and the anti- D-string away from each other. This shows that at $\alpha = 1$ (as well as at other values of α which are not equivalent to $\alpha = 0$) the D-string anti- D-string system is bound.

  χ, $\chi^\dagger$ have half integer mode expansion again at $\alpha = 2$. Following an identical analysis one can easily verify that this time the new ground state $|G'\rangle$ has the same quantum numbers under various projection operators as in the case of $\alpha = 0$ and hence this point is equivalent to $\alpha = 0$. In particular the set of states $\psi^\mu_{-\frac{1}{2}}|G'\rangle$ survive all projections and represent the freedom of separating the D-string - anti-D-string pair away from each other. This also shows that the D-string anti- D-string system is most tightly bound (*i.e.* the mode corresponding to the freedom of separating the pair has maximum mass) at $\alpha = 1$. Thus we might expect that the $\alpha = 1$ point will represent the true minimum of the tachyon potential when we increase the radius of X away from $1/\sqrt{2}$. We shall see this more explicitly in section 4.

- Note also that at $\alpha = 1$, the ground state $|G\rangle$ is invariant under $(-1)^F h$. This would correspond to a tachyonic mode. However, as we shall show now, this state is projected out by the $(-1)^F \Omega$ projection. First note that since this state can be written as $\chi_{1/2}|0\rangle_{\alpha=1}$, it is the image of the state $\chi_{-1/2}|0\rangle$ at $\alpha = 0$. The $(-1)^F$

invariant combination at $\alpha = 0$, as seen from (3.31), is given by

$$\frac{1}{\sqrt{2}}\{\chi_{-\frac{1}{2}}|0\rangle \otimes (\sigma_3 + i\sigma_2) - \chi^\dagger_{-\frac{1}{2}}|0\rangle \otimes (\sigma_3 - i\sigma_2)\}$$
$$= i(\psi_{-\frac{1}{2}}|0\rangle \otimes \sigma_3 + \xi_{-\frac{1}{2}}|0\rangle \otimes \sigma_2)\,. \tag{3.32}$$

The first term on the right hand side of this equation represents the Wilson line of the U(1) gauge field on the D-string along the compact circle, and is known to have the wrong Ω projection [4]. The second state represents a state carrying momentum along x, but also has wrong Ω projection since the Chan Paton factor is anti-symmetric. Since by construction these states are even under $(-1)^F$, we conclude that both are odd under $(-1)^F\Omega$, and hence are projected out. Thus there is no tachyonic mode of the system at $\alpha = 1$.

This analysis also shows us that if we consider an identical configuration of D-string anti- D-string pair in type IIB string theory, then the $\alpha = 1$ state *does* have a tachyonic mode since there is no Ω projection in this case. This tachyonic instability can be traced to the fact that on (infinitely long) D-string anti- D-string pair in type IIB string theory the tachyon represents a complex field on the world volume, and hence the minimum of the potential lies along a circle $T = T_0 e^{i\theta}$ instead of at two points $\pm T_0$. Since $\pi_0(S^1)$ is trivial, the kink solution is no longer topologically stable (although it does represent a solution of the equations of motion) and hence has a tachyonic mode representing decay into the trivial solution $T = T_0$.

- Once we switch on the tachyon vev we break translational invariance along x, and hence there must be a zero mode representing the freedom of translating the soliton along x. The vertex operator for this zero mode is constructed as follows. Let V_x denote the vertex operator associated with this mode. If δ_ϵ denotes the transformation $x \to x + \epsilon$, then, since the translation invariance is broken by the tachyon vev, we would expect,

$$\delta_\epsilon V_T \propto \epsilon V_x\,. \tag{3.33}$$

V_T is the tachyon vertex operator defined in eqs.(3.11)-(3.13). This gives,

$$V_x^{(-1)} = -e^{-\Phi_B}\xi_B \otimes \sigma_1, \qquad V_x^{(0)} = \psi_B\eta_B \otimes \sigma_1\,. \tag{3.34}$$

This represents a marginal operator for all values of tachyon vev α.

- Note that the sectors with Chan Paton factors I and σ_1 contain Fock space states carrying $((-1)^F, (-1)^F h)$ values (1,1) and $(-1, 1)$ respectively, with the SL(2,R) invariant vacuum assigned quantum numbers $(-1, -1)$. On the other hand, after switching on the tachyon vev, the combined spectrum from the sectors with Chan Paton factors $\begin{pmatrix} 1 & \pm 1 \\ \mp 1 & -1 \end{pmatrix}$ contains Fock space states carrying $((-1)^F, (-1)^F h)$ values $(\pm 1, -1)$ *if we continue to assign quantum numbers* $(-1,-1)$ *to the SL(2,R) invariant vacuum.* Thus once we combine the states from all four Chan Paton sectors, all Fock space states are present in the spectrum before projection under Ω.

Besides the modes discussed here, there are other open string states whose one end lie on the (anti-) D-string and the other end lie on the 9-brane. The effect of tachyon vev on the spectrum of these states can be analysed by very similar method, but we shall not do it here.

4. Deforming away from the critical radius

In this section we shall consider the effect of switching on the perturbation that deforms the radius away from the critical radius. In particular we shall show that:

- In the presence of this perturbation the tachyon develops a tadpole except at $\alpha = 0$ and at $\alpha = 1$. Thus these are the only two points which correspond to solutions of equations of motion.
- As we increase the radius of the circle in the x direction, the tachyon represents a state with positive mass^2 if $\alpha = 1$. Thus $\alpha = 1$ represents a stable solution for $R > (1/\sqrt{2})$.
- In the $R \to \infty$ limit we can represent the $\alpha = 1$ solution as a D0-brane in type I string theory.

We begin by analysing the effect of radius deformation on the one point function of the tachyon at an arbitrary value α of the background tachyon field. If V_r denotes the closed string vertex operator associated with the radius deformation, then the effect of switching on this perturbation on the tachyon one point function is proportional to the two point function $\langle V_r V_T \rangle_\alpha$ on the disk, with V_T inserted at the boundary of the disk and V_r inserted in the interior of the disk. For definiteness we shall take V_T in the -1 picture and V_r in the $(-1,0)$ picture for this calculation.[8] $V_T^{(-1)}$ is given in eq. (3.14), whereas using eqs.(3.1)-(3.7) we may express $V_r^{(-1,0)} \propto e^{-\widetilde{\Phi}} \partial X_R \widetilde{\psi}$ as

$$V_r^{(-1,0)} \propto e^{-\widetilde{\Phi}} \eta \Big(e^{i\sqrt{2}(\phi_L + \phi_R)} + e^{i\sqrt{2}(\phi_L - \phi_R)} - e^{-i\sqrt{2}(\phi_L - \phi_R)} - e^{-i\sqrt{2}(\phi_L + \phi_R)} \Big), \qquad (4.1)$$

$\widetilde{\Phi}$ being the left-moving component of the bosonized ghost [20]. The vertex operators $e^{\pm i\sqrt{2}(\phi_L + \phi_R)}$ carry net ϕ momentum. Since neither the tachyon vertex operator nor the

[8] $(-1,0)$ picture means that we use -1 picture for the left-moving sector and 0 picture for the right-moving sector.

background tachyon field carry any ϕ momentum, and since the Neumann boundary condition on ϕ conserves ϕ momentum, the correlation function involving these two terms in $\langle V_r^{(-1,0)} V_T^{(-1)} \rangle_\alpha$ vanish. Thus the matter part of the correlation function is proportional to:

$$\langle \eta(P) \Big(e^{-i\sqrt{2}(\phi_L - \phi_R)} - e^{i\sqrt{2}(\phi_L - \phi_R)} \Big)(P) \eta_B(Q) Tr \Big(\sigma_1 e^{\frac{i}{2\sqrt{2}} \alpha \sigma_1 \oint dt \partial_t \phi_B} \Big) \rangle \,, \tag{4.2}$$

where P denotes a point in the bulk, Q denotes a point on the boundary and t parametrizes the boundary. Note that we have included the effect of the tachyon background by including the exponential factor inside the trace over the Chan Paton factor, so $\langle\ \rangle$ in the above equation represents correlation function in the theory without any tachyon vev.[9] The trace over the Chan Paton factors can be easily evaluated and the contribution may be written as

$$i \langle \eta(P) \eta_B(Q) \Big(e^{i\sqrt{2}(\phi_L - \phi_R)} - e^{-i\sqrt{2}(\phi_L - \phi_R)} \Big)(P) \sin(\frac{1}{2} \alpha \pi w_\phi) \rangle \,, \tag{4.3}$$

where

$$w_\phi = \frac{\sqrt{2}}{2\pi} \oint dt \partial_t \phi_B \,, \tag{4.4}$$

measures the total ϕ winding number carried by the operators in the bulk. Since $e^{\pm i\sqrt{2}(\phi_L - \phi_R)}$ carry ϕ winding number ± 2 we see that this correlation function is proportional to

$$i \sin(\alpha\pi) \langle \eta(P) \eta_B(Q) \Big(e^{-i\sqrt{2}(\phi_L - \phi_R)} + e^{i\sqrt{2}(\phi_L - \phi_R)} \Big)(P) \rangle \,. \tag{4.5}$$

The correlation function $\langle \eta(P) \eta_B(Q) \rangle$ is non-vanishing on the disk. On the other hand, due to Neumann boundary condition on ϕ, the expectation values of $e^{\pm i\sqrt{2}(\phi_L - \phi_R)}(P)$ on the disk are non-vanishing and equal. Thus we see that the one point function of the tachyon in the presence of the radius perturbation is proportional to $\sin(\alpha\pi)$. This vanishes only at $\alpha = 0$ and $\alpha = 1$ (other integer values of α being equivalent to 0 or 1). Thus only at these two values of α we get solutions of the equations of motion.

Next we analyse the two point function of the tachyon field at $\alpha = 1$. This time we can take the two tachyon vertex operators to be in the -1 picture and V_r in the (0,0) picture, given by:

$$\begin{aligned} V_r^{(0,0)} &= \partial X_R \partial X_L = -\frac{1}{2} \xi \eta \tilde{\xi} \tilde{\eta} \\ &= \frac{1}{4} \eta \tilde{\eta} \Big(e^{-i\sqrt{2}(\phi_L - \phi_R)} + e^{i\sqrt{2}(\phi_L - \phi_R)} + e^{-i\sqrt{2}(\phi_L + \phi_R)} + e^{i\sqrt{2}(\phi_L + \phi_R)} \Big)(P) \,. \end{aligned} \tag{4.6}$$

[9] Since the tachyon vertex operator connects a D-string with Z_2 Wilson line to an anti- D-string without a Z_2 Wilson line, there is some subtlety in writing the contribution of the operators at the boundary as a trace over Chan Paton factors. However, since the D-string and the anti- D-string differ in their coupling to the RR sector states, and also in their coupling to the states carrying odd winding number along the x direction (see, for example [11]), this subtlety does not affect the computation when the operator in the bulk is in the NSNS sector and carries zero winding number along x.

Thus the matter part of the relevant correlation function is of the form:

$$\langle\eta(P)\tilde{\eta}(P)\Big(e^{-i\sqrt{2}(\phi_L-\phi_R)}+e^{i\sqrt{2}(\phi_L-\phi_R)}+e^{-i\sqrt{2}(\phi_L+\phi_R)}+e^{i\sqrt{2}(\phi_L+\phi_R)}\Big)(P)$$
$$\eta_B(Q_1)\eta_B(Q_2)Tr\Big(\sigma_1\sigma_1 e^{\frac{i}{2\sqrt{2}}\sigma_1\oint dt\partial_t\phi_B}\Big)\rangle\,, \tag{4.7}$$

where again P denotes a point in the interior of the disk and Q_1 and Q_2 denote points on the boundary of the disk. The trace over the Chan Paton factors gives $\cos(\pi w_\phi/2)$. As in the case of one point function, the contribution to the correlator from the terms involving $\exp(\pm i\sqrt{2}(\phi_L+\phi_R))$ vanishes due to ϕ momentum conservation. For the other two terms carrying $w_\phi=\pm 2$, $\cos(\pi w_\phi/2)$ gives a contribution of -1. Thus the net effect of the tachyon vev corresponding to $\alpha=1$ is to change the sign of this three point function. Since we already know that in the absence of tachyon vev the tachyon mass^2 *decreases* as we increase the radius, we conclude that for $\alpha=1$ the tachyon mass^2 *increases* as we increase the radius. Thus $\alpha=1$ represents a stable ground state of the system for $R>(1/\sqrt{2})$. This analysis also shows that for $R<(1/\sqrt{2})$ the tachyon mass^2 becomes negative at $\alpha=1$, indicating that the spinor state becomes unstable in this range of values of R.

We can generalise this analysis to include arbitrary number of tachyon vertex operators on the boundary and arbitrary number of insertions of $V_r^{(0,0)}$ in the interior. For $\alpha=1$, the effect of the tachyon vev is to introduce a factor of

$$\exp(\frac{i}{2}\pi w_\phi\sigma_1)=\cos(\pi w_\phi/2)+i\sigma_1\sin(\pi w_\phi/2)\,, \tag{4.8}$$

at the boundary. Since each term in $V_r^{(0,0)}$ carries even w_ϕ charge, the total w_ϕ carried by all the closed string vertex operators is always even, and hence the contribution from the $\sin(\pi w_\phi/2)$ term vanishes. The effect of the $\cos(\pi w_\phi/2)$ term is to change the sign of every term in each $V_r^{(0,0)}$ carrying odd $(w_\phi/2)$. Using expression (4.6) for $V_r^{(0,0)}$ we see that it gets transformed to:

$$\frac{1}{4}\eta\tilde{\eta}\Big(-e^{-i\sqrt{2}(\phi_L-\phi_R)}-e^{i\sqrt{2}(\phi_L-\phi_R)}+e^{-i\sqrt{2}(\phi_L+\phi_R)}+e^{i\sqrt{2}(\phi_L+\phi_R)}\Big)(P)$$
$$=-\frac{1}{2}\eta\tilde{\eta}\psi\tilde{\psi}=-\partial\phi'_R\partial\phi'_L\,. \tag{4.9}$$

Thus tachyon vev corresponding to $\alpha=1$ transforms the vertex operator $\partial X_R\partial X_L$ to $-\partial\phi'_R\partial\phi'_L$. In other words, increasing the radius of the x coordinate in the presence of the tachyon vev corresponds to decreasing the radius of ϕ' coordinate with no tachyon vev. If the x radius is scaled up by a factor of L, this would correspond to scaling down the ϕ' radius by a factor of L. Thus an x radius of $L/\sqrt{2}$ will correspond to a ϕ' radius of $1/(\sqrt{2}L)$. Since the tachyon vertex operator carries ϕ' momentum (as can be easily verified from eqs.(3.4) and (3.14)) a decrease in the ϕ' radius will increase the mass^2 of the tachyon. This confirms our previous perturbative result.

The analysis can be easily extended to all NS sector vertex operators with Chan Paton factors proportional to σ_1 or I. These Chan Paton factors commute with the

Chan Paton factors associated with the tachyon. Furthermore, since for correlation functions involving NS sector states on the disk we have Neumann boundary condition on ϕ ($\phi_L = \phi_R$) everywhere on the boundary, the vertex operators inserted at the boundary may carry ϕ momentum but no ϕ winding. Hence insertion of these vertex operators do not create any discontinuity in ϕ_B on the boundary, and $\oint \partial_t \phi_B$ along the boundary can be identified as $\sqrt{2}\pi w_\phi$, with w_ϕ measuring the total ϕ winding number of all the operators inserted in the interior of the disk. Thus the effect of a tachyon vev corresponding to $\alpha = 1$ may again be represented by (4.8). We can now repeat the argument of the previous paragraph to show that the effect of increasing the radius of the x coordinate in the presence of the tachyon vev is to effectively decrease the radius of the ϕ' coordinate.

Next we consider the case where on the boundary we have vertex operators with Chan Paton factors proportional to $\sigma^\pm = \sigma_3 \pm i\sigma_2$. For illustration we consider the case where we have two vertex operators at the boundary – one proportional to σ^+ and the other to σ^- – but this construction can be easily generalized for any correlation function with arbitrary insertions of NS sector open string vertex operators at the boundary. For the two point correlator the typical operator insertions at the boundary take the form:

$$Tr\Big(\sigma^- e^{ik_1\phi_B(Q_1)} e^{\frac{i}{2\sqrt{2}}\sigma_1 \int_{Q_1}^{Q_2} \partial_t\phi_B dt} \sigma^+ e^{ik_2\phi_B(Q_2)} e^{\frac{i}{2\sqrt{2}}\sigma_1 \int_{Q_2}^{Q_1+2\pi} \partial_t\phi_B dt}\Big)\,, \tag{4.10}$$

where t denotes the coordinate on the boundary of the disk with periodicity 2π, Q_1 and Q_2 represent the points on the boundary where the vertex operators have been inserted, and k_1 and k_2 are the ϕ momenta carried by the vertex operators. Using the commutation relations between the Pauli matrices, and the relations

$$\begin{aligned}\int_{Q_1}^{Q_2} \partial_t\phi_B dt &= \phi_B(Q_2) - \phi_B(Q_1)\,,\\ \int_{Q_2}^{Q_1+2\pi} \partial_t\phi_B dt &= \phi_B(Q_1) - \phi_B(Q_2) + \sqrt{2}\pi w_\phi\,,\end{aligned} \tag{4.11}$$

one can bring (4.10) to the form:

$$Tr\Big(\sigma^- e^{i(k_1+\frac{1}{\sqrt{2}})\phi_B(Q_1)} \sigma^+ e^{i(k_2-\frac{1}{\sqrt{2}})\phi_B(Q_2)} e^{i\pi w_\phi \sigma_1/2}\Big)\,. \tag{4.12}$$

Thus we see that the effect of switching on the tachyon vev is to shift the ϕ momenta in vertex operators associated with $\sigma^\mp$ by $\pm 1/\sqrt{2}$ – a fact which has already been taken into account in section 3 – and simultaneously introduce a factor of $\exp(i\pi w_\phi \sigma_1/2)$ at the boundary. The later factor transforms the operators $\partial X_R \partial X_L$ inserted in the interior of the disk to $-\partial\phi'_R\partial\phi'_L$ as before. This shows that for all correlation functions involving NS sector open string vertex operators, the effect of deforming the x-radius after switching on the tachyon vev is to change the ϕ' radius in the opposite direction.

From (3.34), and the bosonisation rules (3.7) we see that the vertex operator V_x in the zero picture is proportional to

$$i\partial\phi'_B \otimes \sigma_1\,, \tag{4.13}$$

and hence represents a Wilson line along the ϕ' coordinate. This clearly remains a marginal deformation even when we change the ϕ' radius $R_{\phi'}$. This shows that when we change the radius of the x coordinate in the presence of tachyon vev, the translational zero mode continues to represent a marginal deformation. Furthermore, this zero mode, being a Wilson line along ϕ', represents a periodic coordinate with periodicity proportional to $1/R_{\phi'}$. Since $R_{\phi'}$ is proportional to $(1/R)$, R being the radius of the x coordinate, we see that the periodicity is proportional to R, as is expected of a zero mode representing translation along a circle of radius R. In particular, this shows that in the $R \to \infty$ limit the zero mode takes value on a real line, representing the position of the soliton along the x axis.

In calculating the spectrum in the $R \to \infty$ limit we can simplify the analysis by making a T-duality transformation in the coordinate ϕ'. Since (after taking into account the effect of the tachyon vev) the coordinate ϕ' has radius $1/(2R)$, the T-duality will give rise to dual coordinate ϕ'_D of radius $2R$, and at the same time transform the Neumann boundary condition on ϕ' to Dirichlet boundary condition on ϕ'_D. In the limit $R \to \infty$, ϕ'_D has infinite radius, and we recover an extra SO(9) symmetry that mixes the pair (ϕ'_D, ξ) with (X^μ, ψ^μ) for $1 \le \mu \le 8$.[10] Thus we see that although to start with we had broken the SO(9) symmetry of the problem by identifying one special direction along which the D-string anti- D-string pair lies, we recover the full SO(9) symmetry of the spectrum at the end.

Next we turn to the fate of the Ramond sector states under radius deformation. We shall focus on two point function involving two Ramond sector vertex operators; this is sufficient for analysing the spectrum. Since the insertion of a Ramond sector vertex operators converts a Neumann boundary condition on ϕ to a Dirichlet boundary condition, we can no longer represent the tachyon vertex operator in the zero picture by (3.15) everywhere in the boundary. However we can still express it as

$$V_T^{(0)} = i\sqrt{2}\partial\phi_R \otimes \sigma_1 \, . \tag{4.14}$$

Since the Ramond sector vertex operators do not carry either momentum or winding along ϕ, ϕ_R is continuous across the Ramond sector vertex operators inserted at the

[10] Although $(-1)^F$ and $h : X \to X + \sqrt{2}\pi$ projections act differently on (ϕ'_D, ξ) and (X^μ, ψ^μ), from the analysis of the previous section we have seen that once we combine the states from all four Chan Paton sectors, we have all the Fock space states carrying $(-1)^F = \pm 1$ and $h = \pm 1$. Thus the full spectrum is invariant under the exchange $(\phi'_D, \xi) \leftrightarrow (X^\mu, \psi^\mu)$ before Ω projection. On the other hand, one can show by a detailed analysis that the Ω projection also preserves the SO(9) symmetry of the spectrum. The main point is to note that Ω induces a transformation $\phi' \to -\phi'$, since otherwise the vertex operator $V_x^{(0)} \propto \partial_t \phi'_B \otimes \sigma_1$, involving tangential derivative of ϕ' along the boundary, could not be Ω invariant. This in turn implies that the dual coordinate ϕ'_D does not change sign under Ω, just as the X^μ's for $1 \le \mu \le 8$. Furthermore, since the vertex operators $\xi_B \otimes \sigma_1$ and $\psi^\mu \otimes I$ are both present in the spectrum in the -1 picture even after the Ω projection, the oscillators of ξ and ψ^μ transform under Ω in identical manner.

boundary. Following the same arguments as in the case of NS sector states, we see that the effect of switching on the tachyon vev on the correlation functions is to

- give a factor of
$$\exp\Big(\frac{i}{\sqrt{2}}\sigma_1 \oint dt\partial_t\phi_R\Big), \tag{4.15}$$
at the boundary, and

- introduce factors of
$$\exp(\mp i\sqrt{2}\phi_R), \tag{4.16}$$
at the location of Chan Paton factors $\sigma^\pm$. However, since the Ramond sector vertex operators are localized at $\phi = 0$, (4.16) becomes trivial.

Let us now switch on the radius deformation. This corresponds to adding a term proportional to integral of $\partial X_L \partial X_R$ to the two dimensional action, which in turn can be expressed as (4.6). Using the operator product expansion of scalar fields, it is easy to see that acting on the right hand side of (4.6), (4.15) gives an overall factor of -1. Thus for this correlation function, the effect of the tachyon vev is to change the sign of the $\partial X_L \partial X_R$ term, *i.e.* to convert a perturbation corresponding to an increase in radius to one corresponding to a decrease in the radius. In particular in order to compute the spectrum of Ramond sector states at x radius $R = L/\sqrt{2}$ in the presence of a tachyon vev $\alpha = 1$, we simply need to compute the spectrum at a new x radius $R_x = 1/(\sqrt{2}L)$ in the absence of any tachyon vev. If we T-dualize in the x coordinate, and use the dual radius x_D as the independent bosonic variable, then the x_D radius goes to ∞ as $R \to \infty$, and the Neumann boundary condition on x corresponds to Dirichlet boundary condition on x_D. Thus we recover the full SO(9) invariance of the spectrum in this limit.[11]

We can now address the question of the dependence of the mass of the soliton on the radius. In the absence of tachyon vev, the mass of the D-string anti- D-string pair is expected to be proportional to R. Let us first see how this arises in the string calculation. For this we compute the interaction between a pair of such systems at large separation by computing the partition function of open strings with two ends lying on

[11] Again in this case we need to ensure that various projections do not destroy the symmetry under the exchange of x_D with one of the transverse coordinates x^μ. To see this, let us first note that in this case the h projection simply removes all states in Chan Paton sectors σ_1 and σ_2, since states in this sector carry odd unit of x momentum, and hence become infinitely massive in the limit when the effective x-radius goes to zero. Thus we are left with Chan Paton factors I and σ_3. The action of Ω on Ramond sector states with Chan Paton factors $I \pm \sigma_3$, representing wrapped D-branes and anti-D-branes respectively, can be shown to differ by a factor of -1 using the results of [21]. Thus when we combine the set of Fock space states from both sectors, we can simply ignore the Ω projection. Finally, since $(-1)^F$ changes the sign of ψ^μ as well as of ψ, projection under $(-1)^F$ does not destroy SO(9) invariance.

the two solitons, and equate it to

$$G_N M^2/l^6\,, \tag{4.17}$$

where G_N is the Newton's constant, M is the mass of the system, and l is the separation. G_N in the (8+1) dimensional theory obtained by compactifying the x direction is proportional to $(1/R)$. On the other hand, for large separation l between the two systems, the leading contribution to the open string partition function comes from the states with large x momentum. The density of such states is proportional to R. Thus we get, $(M^2/R) \propto R$. This gives $M \propto R$.

In the presence of tachyon vev, G_N is still proportional to $(1/R)$. The open string states in NS (R) sector are now better regarded as states carrying definite momentum along ϕ' (x), with ϕ' (x) having radius proportional to $1/R$. For large l the leading contribution comes from states with large ϕ' (x) momentum whose density of states is proportional to $R_{\phi'}$ (R_x) and hence to R^{-1}. Thus we get $(M^2/R) \propto (1/R)$. This shows that M^2 is independent of R as has been advertised earlier.

Let us now summarise the main result of this section. Starting from the description of the SO(32) spinor of type I as a tachyonic kink solution on the D-string anti- D-string pair, we have shown that there is a description of these states in terms of an exact boundary conformal field theory. In this description these states are regarded as non-supersymmetric D0-branes of type I string theory. There is no GSO projection on the open string states[12]. The NS sector ground state of the open string represents a tachyonic mode, but this is odd under the world sheet parity transformation Ω and is projected out. Finally, the extra factor of $\sqrt{2}$ in the mass formula (eq. (2.18)) compared to the mass of an ordinary D0-brane can be understood as follows. The open string partition function of an ordinary D0-brane contains a projection operator $\frac{1}{2}(1 + (-1)^F)$; of this the first term represents NSNS exchange diagram, whereas the second term represents the RR exchange diagram in the closed string channel. In the absence of this projection operator the NSNS exchange diagram doubles, whereas the RR exchange diagram is absent. This gives a net enhancement factor of $\sqrt{2}$ in the mass formula.[13]

5. Other solitons on brane-antibrane pair

Encouraged by the fact that the tachyonic kink solution on the D-string anti- D-string pair makes sense, we can try to look for other kinds of solitons on the brane anti-brane pair. The first example that we shall study is a vortex like solution on the membrane anti-membrane pair in type IIA string theory. In this case the tachyon associated with

[12]The possibility of existence of such D0-branes was first mentioned by Bergman [22].

[13]The mass is being measured in the variable of the type IIB theory before the Ω projection; hence in this calculation we do not take into account the extra factor of $1/2$ coming from the Ω projection.

the open string stretched between the membrane and the anti-membrane is a complex scalar field T, T and T^* being associated with the two possible orientations of the open string. Thus the manifold describing the minimum of the tachyon potential has the structure of a circle, described by the equation:

$$|T| = T_0\,, \tag{5.1}$$

for some real number T_0. The argument of [6] shows that at $|T| = T_0$ the negative contribution from the tachyon potential exactly cancels the sum of the tensions of the membrane and the anti-membrane, giving vanishing total energy density.

There is a $U(1) \times U(1)$ gauge field living on the world volume of the membrane anti-membrane system. The tachyon carries one unit of charge under each of these gauge fields. If $A^{(1)}_\mu$ and $A^{(2)}_\mu$ denote the gauge fields coming from the D-brane and the anti- D-brane respectively, then the kinetic term for the tachyon field takes the form:

$$|D_\mu T|^2\,, \tag{5.2}$$

where

$$D_\mu T = (\partial_\mu - iA^{(1)}_\mu + iA^{(2)}_\mu)T\,. \tag{5.3}$$

We can now consider a static, finite energy vortex like configuration for the tachyon field such that in the polar coordinates (r, θ) on the membrane, the asymptotic field configuration takes the form:

$$T \simeq T_0 e^{i\theta}, \qquad A^{(1)}_\theta - A^{(2)}_\theta \simeq 1\,, \qquad \text{as} \quad r \to \infty\,, \tag{5.4}$$

so that both the kinetic and the potential terms vanish sufficiently fast as $r \to \infty$. This soliton will describe a stable, finite mass particle in type IIA string theory.

In order to identify this particle, let us note that for this solution,

$$\oint (A^{(1)} - A^{(2)}) \cdot dl = 2\pi\,, \tag{5.5}$$

and hence it carries one unit of magnetic flux associated with the gauge field $(A^{(1)}_\mu - A^{(2)}_\mu)$ on the world volume of the membrane anti-membrane system. This in turn implies that it carries one unit of $D0$ brane charge. Thus this soliton is indistinguishible from the D0-brane of type IIA string theory, and is simply a different representation of the same particle. Presumably this relation can be proved explicitly by studying the boundary conformal field theory describing this solution in a manner analogous to the previous two sections. The above construction can be trivially generalised to represent the p-brane of type II string theory as a vortex solution on the $(p+2)$-brane anti- $(p+2)$-brane pair.

The second example that we shall study is a string like solution on the 5-brane anti-5-brane system in type I string theory. The gauge field living on the world volume of the system is $SU(2) \times SU(2)$ [23, 21], and the tachyon associated with the open string stretched between the brane and the anti-brane is a 2×2 matrix valued field transforming in the $(2, 2)$ representation of the $SU(2) \times SU(2)$ group. Thus if U and V denote the SU(2) matrices associated with the two gauge groups, then

$$T \to UTV^\dagger \,, \tag{5.6}$$

under a gauge transformation. The projection under the world-sheet parity transformation Ω reduces the number of real fields from 8 to 4, giving the following restriction on the form of T:

$$T = \lambda W \,, \tag{5.7}$$

where λ is a real number and W is a 2×2 unitary matrix. The manifold labelling the minimum of the tachyon potential is described by the SU(2)×SU(2) invariant equation,

$$\det(T) = \lambda_0^2 \,, \tag{5.8}$$

for some constant λ_0. This gives

$$\lambda = \lambda_0 \,. \tag{5.9}$$

Thus this manifold is labelled by the SU(2) matrix W, and has the structure of the SU(2) group manifold, namely S^3. By the result of ref. [6], on this manifold the total contribution to the energy density from the tension of the brane and the anti-brane, and the tachyon potential vanishes.

The kinetic energy of the tachyon field has the form:

$$Tr((D_\mu T)^\dagger (D^\mu T)) \,, \tag{5.10}$$

where

$$D_\mu T = \partial_\mu T + i A_\mu^{(1)} T - i T A_\mu^{(2)} \,, \tag{5.11}$$

$A_\mu^{(1)}$ and $A_\mu^{(2)}$ being the SU(2) gauge fields living on the brane and the anti-brane. Since a string like configuration in five dimensions has asymptotic boundary S^3, we can get a finite energy, static, string like solution in this (5+1) dimensional field theory by imposing the following asymptotic form for various fields:

$$T \simeq \lambda_0 U, \qquad A_\mu^{(2)} \simeq 0, \qquad A_\mu^{(1)} \simeq i \partial_\mu U U^{-1} \,, \tag{5.12}$$

where U is an SU(2) matrix valued function, corresponding to the identity map (with unit winding number) from the asymptotic boundary S^3 to the SU(2) group manifold S^3. This makes both the kinetic and the potential term vanish sufficiently rapidly at infinity so as to give a finite energy configuration. Non-triviality of $\pi_3(S^3)$ guarantees topological stability of this solution.

In order to identify this solution, we note from the asymptotic form of $A_\mu^{(1)}$ that it carries one unit of instanton number for the gauge field $A_\mu^{(1)}$ living on the five brane. Since this is a source of the D-string charge in type I string theory [24] we see that the soliton solution that we have constructed can be identified to a D-string of type I string theory.

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